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Punctiform charge, Maxwell's equations and the meaning of the displacement current

Carga puntiforme, equações de Maxwell e o significado da corrente de deslocamento

Carga puntiforme, ecuaciones de Maxwell y el significado de la corriente de desplazamiento

Paulo Cesar Facin[1]

ABSTRACT

This paper considers the case of a punctiform charge displacement in relation to the laboratory reference in constant speed, the electric and magnetic fields given by the electromagnetic four-potential Lorentz transformation. By assuming a limit tending to zero for the charge volume, we demonstrated that, as expected, this choice for the fields respects perfectly the four Maxwell's equations. In addition, to our surprise, the displacement current term generalizes the current density term and makes the Ampère-Maxwell's law more symmetric when compared to the Faraday's law. Therefore, we could notice that the discrete nature of this problem allowed us to capture the electric field variation with time, which is fundamental for the understanding of the displacement current and the Ampère-Maxwell's law.

Keywords: Punctiform charge; Displacement current; Maxwell's equations.

RESUMO

Consideramos o caso de uma carga puntiforme se deslocando em relação ao referencial do laboratório com velocidade constante, assumimos os campos elétrico e magnético dados pela transformação de Lorentz do quadri-potencial eletromagnético. Assumindo um limite que tende a zero para o volume da carga mostramos que, como era esperado, essa escolha para os campos respeita perfeitamente as quatro equações de Maxwell, além disso, para nossa surpresa, o termo da corrente de deslocamento generaliza o termo de densidade de corrente e torna a lei de Ampère-Maxwell mais simétrica em relação a lei de Faraday. Percebemos então que a natureza discreta desse problema possibilitou capturar a variação do campo elétrico com o tempo que é fundamental para o entendimento da corrente de deslocamento e a lei de Ampère-Maxwell.

Palavras-chave: *Carga puntiforme; Corrente de deslocamento; Equações de Maxwell.*

RESUMEN

Consideramos el caso de una carga puntiforme que se mueve en relación al referencial del laboratorio con velocidad constante, asumimos que los campos eléctrico y magnético dados por la transformación de Lorenz del cuadri-potencial electromagnético. Tomando un límite tendiendo a cero para el volumen de la carga, mostramos que, como era esperado, esta elección para los campos cumple perfectamente las cuatro ecuaciones de Maxwell, además, para nuestra sorpresa, el término de la corriente de desplazamiento generaliza el termino de densidad de corriente y resulta la ley de Ampère-Maxwell más simétrica en relación a ley de Faraday. De esta forma, pudimos percibir que la naturaleza discreta de este problema, nos proveo capturar la variación del campo eléctrico con el tiempo, que es fundamental para la comprensión de la corriente de desplazamiento y la ley de Ampère-Maxwell.

Palabras clave: *Carga puntiforme; Corriente de desplazamiento; Ecuaciones de Maxwell.*

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1. INTRODUCTION

The subject of "displacement current" is one of those subjects in the discipline of physics that we are used to accepting. The arguments given by J. C. Maxwell for the existence of such a term in the Ampère equation are not so difficult to understand and accept. Furthermore, in favor of this acceptance the displacement current completes the interpretation of light as an electromagnetic wave. In many texts, the explanations that give us the condition of acceptance of the arguments for the existence of the displacement current are remarkable, but we are a little far from the more intuitive meaning of this term.

The displacement current is usually approached with the charge process of a flat and parallel plates capacitor. In the term proposed by Maxwell, the displacement current, is introduced to solve the problem of the Ampère's law when the current is not stationary, that is, when $\vec{\nabla} \cdot \vec{J} \neq 0$, because the divergent of a rotational (of the magnetic field) must be null. Maxwell suggests a new term $\varepsilon_0 \frac{\partial \vec{E}}{\partial t}$, that is, a term that does not require the presence of currents at the point where the magnetic field rotational is calculated, in addition, the new term helps the charge conservation. Although textbooks and articles on the topic present this strategy to introduce the displacement current term in the Ampère's law, it is not very intuitive for students and is perceived as quite difficult, remaining "a stone in the shoe" in the education regarding Maxwell's electromagnetism. The study put forward by John Roche[2] is an attempt to clarify Maxwell's contribution to the Ampère's law and sheds some light on the topic, showing the historical difficulty that many scientists face in the interpretation and acceptance of this term. In John Roche's study, more emphasis is placed on obtaining the magnetic field between the plates of a capacitor. That author suggests that this magnetic field is due to three sources, the wire conducting the current that provides the plates with charge, the currents in the plates, and Maxwell's term due to the variation in the electric field time between the plates. The author also cites the Cullwick's paradox[3], which calculated the magnetic field created by a punctiform charge in uniform movement using the Biot-Savart's law and also by the displacement current term. According to John Roche, the paradox appears when Cullwick sums the two values and obtains twice the value set for the local magnetic field. Then, Cullwick suggests, in an attempt to solve the paradox, that the two terms cannot be used together. More recently, this subject was addressed by John W. Arthur[4], who presented and answered nine more elementary questions regarding the displacement current. The author uses the integral form of the Ampère-Maxwell's law and shows that the choice of the Coulomb's and Biot-Savart's fields, for a punctiform charge with uniform speed, respects the Ampère-Maxwell's law. The study shows that the Biot-Savart's law produces a magnetic field that does not satisfy the Ampère's law, showing that the integral of the magnetic field contributions on a circumference depend on the distance of the circumference to the charge and not only the current that crosses the circumference. In this work by J. W. Arthur, for the chosen fields to respect Ampère's law, it is suggested that we have to consider that not only the charge crosses the area limited by the circumference, but also the displacement current crosses it. However, the charge is limited to an area of radius α, and the displacement current is limited to the area that excludes the charge. The integral of the currents is then carried out in two parts, one in the limits of zero up to the value of the charge ray "α" representing the 'true' current contribution and the other from α to r, representing the

[2] [ROC98]
[3] [CUL16]
[4][ART09]

displacement current contribution, and the sum of these two currents is what satisfies the Ampère's law.

In this study, we used a differential form of the Maxwell's equations and treated the uniform movement of a punctiform charge with uniform speed. We observed that in the non-relativistic case, the Coulomb's and Biot-Savart's laws satisfy the Ampère-Maxwell's law, but do not satisfy the Faraday's law. However, when using the relativistic transformations for the fields, the Faraday's law is also satisfied. In addition, we found that, surprisingly, in both cases, the displacement current term generalizes the current density term, or charge current.

The fields ascribed to a punctiform charge are important due to the Superposition Principles that allows the calculation of more complex fields as a sum of the fields of continuous / discrete distributions of punctiform charges.

2. Punctiform Charge Fields

Let's then analyze the electric and magnetic fields, in vacuum, in a fixed-point P in relation to the laboratory reference (without the line) due to the presence of a punctiform charge q dislocating at a speed v_x in relation to that point. Someone at the reference origin at the laboratory notices the charge moving away towards the direction x at a speed v_x, and the fields in P located by the vector $\vec{r}$, as in Figure 1:

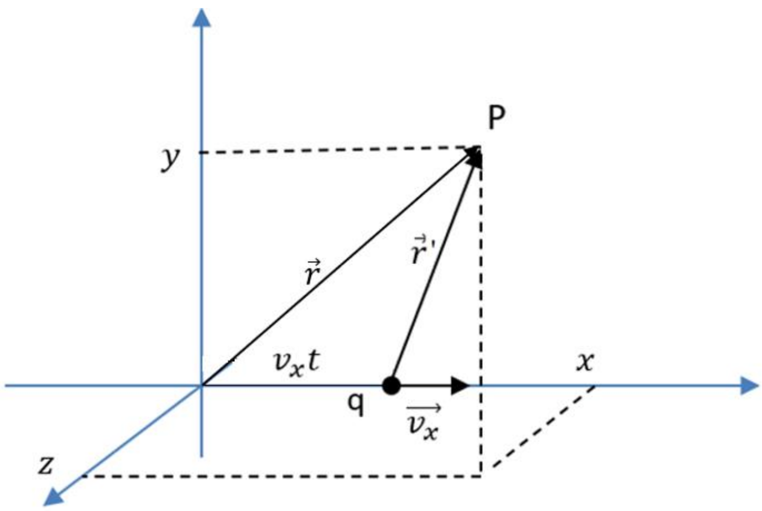

Figure 1 - Punctiform charge q in uniform movement at speed v_x in relation to the laboratory reference system. At point P, Maxwell's equations are analyzed.

An observer at the charge reference observes a Coulomb electric field $\vec{E}'$ around it, including at point P:

$$\vec{E}'(\vec{r}') = \frac{\mu_0 c^2}{4\pi}\frac{q}{r'^2}\hat{r}' \quad \text{and} \quad \vec{B}'(\vec{r}') = 0 \quad , \tag{1}$$

where the constant μ_0 is the vacuum magnetic permeability and is related to the vacuum electric permittivity through the light speed in the vacuum $\varepsilon_0\mu_0 = \frac{1}{c^2}$.

However, in the laboratory reference, this field must be changed according to the Lorentz four-potential transformations, where the scalar potential is $A^0 = \emptyset$, and the vector potential present the components cA^1, cA^2, cA^3, where c is the light speed in the vacuum. For the four-potential choice, Maxwell's equations keep the same form in both references, that is, we propose that these equations are physics laws, satisfying Einstein's first postulate of the special relativity theory. The fields in the laboratory reference are then obtained as a function of the potentials:

$$\vec{E} = -\vec{\nabla}\emptyset - \frac{\partial \vec{A}}{c\partial t} \tag{2}$$

$$\vec{B} = \vec{\nabla} \times \vec{A} \tag{3}$$

The Lorentz four-potential transformations and the fields can be found in detail in Landau and Lifischtz[5], $\emptyset = \gamma\emptyset'$ and $\vec{A} = \emptyset\frac{\vec{v}_x}{c^2}$. Bearing in mind that the Lorentz factor is $\gamma = \frac{1}{\sqrt{1-\frac{{v_x}^2}{c^2}}}$ and the charge is an invariant, we obtain:

$$\emptyset' = \frac{\mu_0 c^2}{4\pi}\frac{q}{r'} \tag{4}$$

$$\emptyset = \gamma\emptyset' = \frac{\mu_0 c^2}{4\pi}\gamma\frac{q}{r'} \tag{5}$$

The observer at the charge reference (with line) notices point P at a speed $-v_x$, and assuming that in the time instant $t = 0$ the origin of the references coincides, for point P on the right of the charge $(x - v_x t > 0)$, we obtain:

$$x' = \gamma(x - v_x t)\ , \quad y' = y\ , \quad z' = z \tag{6}$$

$$r'^2 = \gamma^2(x - v_x t)^2 + (y^2 + z^2) \tag{7}$$

$$\emptyset = \gamma\frac{\mu_0 c^2}{4\pi}\frac{q}{\sqrt{\gamma^2(x-v_x t)^2+(y^2+z^2)}} \tag{8}$$

$$\vec{A} = \gamma\frac{\mu_0}{4\pi}\frac{q\vec{v}_x}{\sqrt{\gamma^2(x-v_x t)^2+(y^2+z^2)}} \tag{9}$$

The expressions for electric and magnetic fields at point P are given by equations (2) and (3), that is:

$$\vec{E}(x,y,z;t) = \frac{\mu_0 c^2 q}{4\pi\gamma^2\left[(x-v_x t)^2+\frac{1}{\gamma^2}(y^2+z^2)\right]^{3/2}}\left[(x - v_x t)\hat{\imath} + y\hat{\jmath} + z\hat{k}\right] \quad , \tag{10}$$

$$\vec{B}(x,y,z;t) = \vec{\nabla} \times \vec{A} = \frac{\vec{v}_x \times \vec{E}(x,y,z;t)}{c^2} \quad , \tag{11}$$

$$\vec{B}(x,y,z;t) = \frac{v_x \mu_0 q}{4\pi\gamma^2\left[(x-v_x t)^2+\frac{1}{\gamma^2}(y^2+z^2)\right]^{3/2}}\left[-z\hat{\jmath} + y\hat{k}\right] \quad . \tag{12}$$

[5] [LAN47]

2.1 GAUSS'S LAW

The analysis now considers the fields given by equations (10) and (12), the divergence of these fields at point P.

$$\frac{\partial E_x}{\partial x} = -\frac{3}{4}\frac{\mu_0 c^2 q (x - v_x t)^2}{\pi\gamma^2\left[(x-v_x t)^2+\frac{1}{\gamma^2}(y^2+z^2)\right]^{5/2}} + \frac{1}{4}\frac{\mu_0 c^2 q}{\pi\gamma^2\left[(x-v_x t)^2+\frac{1}{\gamma^2}(y^2+z^2)\right]^{3/2}} \qquad (13)$$

$$\frac{\partial E_y}{\partial y} = -\frac{3}{4}\frac{\mu_0 c^2 q y^2}{\pi\gamma^4\left[(x-v_x t)^2+\frac{1}{\gamma^2}(y^2+z^2)\right]^{5/2}} + \frac{1}{4}\frac{\mu_0 c^2 q}{\pi\gamma^2\left[(x-v_x t)^2+\frac{1}{\gamma^2}(y^2+z^2)\right]^{3/2}} \qquad (14)$$

$$\frac{\partial E_z}{\partial z} = -\frac{3}{4}\frac{\mu_0 c^2 q z^2}{\pi\gamma^4\left[(x-v_x t)^2+\frac{1}{\gamma^2}(y^2+z^2)\right]^{5/2}} + \frac{1}{4}\frac{\mu_0 c^2 q}{\pi\gamma^2\left[(x-v_x t)^2+\frac{1}{\gamma^2}(y^2+z^2)\right]^{3/2}} \qquad (15)$$

For $x \neq v_x t$, the center of the electric field (charge) is out of the infinitesimal volume that involves point P, resulting in a null flux on the surface that involves that volume, with that, the sum of the terms given by equations (13), (14) and (15) result in:

$$\vec{\nabla}\cdot\vec{\mathrm{E}} = 0 \quad , \qquad x \neq v_x t \qquad (16)$$

Now, for the limit $x \to v_x t$ and $y = z = 0$, a problem of indetermination appears in the electric field and, consequently, we cannot simply carry out the derivates on the field components.

To calculate the divergence at the point x, y, z, which includes the charge, the flux on a volume that involves the charge but does not include it within the volume is usually calculated, such as in a spheric shell in the three-dimension case, or a ring-like biscuit in the two-dimension case, Figure 2.

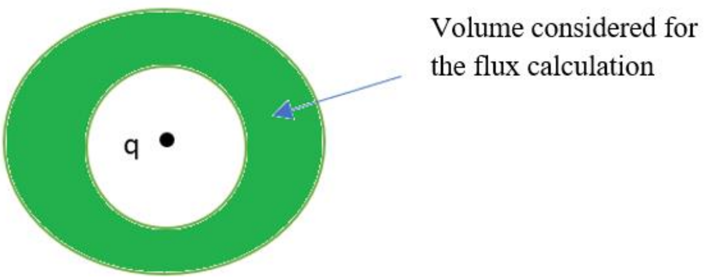

Figure 2 - Volume of the "biscuit" that is considered for the electric field flux calculation in a description of the integral form of the Gauss's law. When this volume tends to zero, the field divergence is obtained.

Let's use the same ideas as the "biscuit", but considering the elements of differential volumes. In two dimensions, we have the interest volumes (which surround the volume that contains point P) as represented in Figure 3.

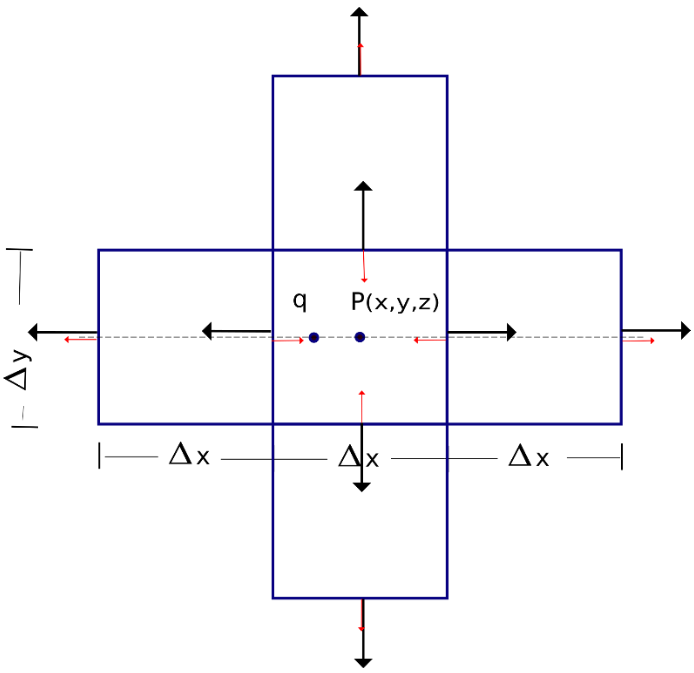

Figure 3 - Volume element in the two-dimension: scheme for the calculation of the electric field divergence of the punctiform charge **q** getting closer to the position **P**. The black arrows represent the electric field vectors, while the red arrows represent the normal vectors of the considered surface.

The position of the charge q is internal to the central volume element, which is centered at point P, but is outside the volume considered, and therefore the flux on this volume must be null. Bearing in mind that the flux is positive when the field is parallel to the normal vector of the surface and negative when it is antiparallel, and simplifying the notation with $E\left(x+\frac{\Delta_x}{2},y,z\right)=E\left(x+\frac{\Delta_x}{2}\right)$, we obtain:

$$\left\{E\left(x+\frac{3\Delta_x}{2}\right)+E\left(x-\frac{3\Delta_x}{2}\right)+\left[-E\left(x+\frac{\Delta_x}{2}\right)-E\left(x-\frac{\Delta_x}{2}\right)\right]\right\}\Delta_y\Delta_z+$$
$$\left\{E\left(y+\frac{3\Delta_y}{2}\right)+E\left(y-\frac{3\Delta_y}{2}\right)+\left[-E\left(y+\frac{\Delta_y}{2}\right)-E\left(y-\frac{\Delta_y}{2}\right)\right]\right\}\Delta_x\Delta_z+$$
$$\left\{E\left(z+\frac{3\Delta_z}{2}\right)+E\left(z-\frac{3\Delta_z}{2}\right)+\left[-E\left(z+\frac{\Delta_z}{2}\right)-E\left(z-\frac{\Delta_z}{2}\right)\right]\right\}\Delta_y\Delta_x=0\ , \tag{17}$$

or, rewriting it,

$$\left[E\left(x+\frac{\Delta_x}{2}\right)+E\left(x-\frac{\Delta_x}{2}\right)\right]\Delta_y\Delta_z+\left[E\left(y+\frac{\Delta_y}{2}\right)+E\left(y-\frac{\Delta_y}{2}\right)\right]\Delta_x\Delta_z+\left[E\left(z+\frac{\Delta_z}{2}\right)+E\left(z-\frac{\Delta_z}{2}\right)\right]\Delta_y\Delta_x=$$

$$\left[E\left(x+\frac{3\Delta_x}{2}\right)+E\left(x-\frac{3\Delta_x}{2}\right)\right]\Delta_y\Delta_z+\left[E\left(y+\frac{3\Delta_y}{2}\right)+E\left(y-\frac{3\Delta_y}{2}\right)\right]\Delta_x\Delta_z+\left[E\left(z+\frac{3\Delta_z}{2}\right)+E\left(z-\frac{3\Delta_z}{2}\right)\right]\Delta_y\Delta_x\ .$$

The term on the left of the equality is the flux on the internal surface of the volume of interest:

$$\left[E\left(x+\frac{\Delta_x}{2}\right)+E\left(x-\frac{\Delta_x}{2}\right)\right]\Delta_y\Delta_z+\left[E\left(y+\frac{\Delta_y}{2}\right)+E\left(y-\frac{\Delta_y}{2}\right)\right]\Delta_x\Delta_z+\left[E\left(z+\frac{\Delta_z}{2}\right)+E\left(z-\frac{\Delta_z}{2}\right)\right]\Delta_x\Delta_y \quad . \tag{18}$$

We could observe that for the charge to reach point P, the point must present coordinates $y = z = 0$ and in the limit that $\Delta_y \to 0$ and $\Delta_z \to 0$, we have $E\left(0 \pm \frac{\Delta_y}{2}\right) = E\left(0 \pm \frac{\Delta_z}{2}\right) = 0$.

Then, only component x survives in the accounting of Equation (18):

$$\frac{\mu_0 c^2 q}{4\pi\gamma^2}\left\{\frac{x-v_x t}{\left[(x-v_x t)^2+\Delta_x(x-v_x t)+\left(\frac{\Delta x}{2}\right)^2\right]^{\frac{3}{2}}}+\frac{\frac{\Delta x}{2}}{\left[(x-v_x t)^2+\Delta_x(x-v_x t)+\left(\frac{\Delta x}{2}\right)^2\right]^{3/2}}+\frac{x-v_x t}{\left[(x-v_x t)^2-\Delta_x(x-v_x t)+\left(\frac{\Delta x}{2}\right)^2\right]^{\frac{3}{2}}}-\frac{\frac{\Delta x}{2}}{\left[(x-v_x t)^2-\Delta_x(x-v_x t)+\left(\frac{\Delta x}{2}\right)^2\right]^{\frac{3}{2}}}\right\}\Delta_y\Delta_z \quad . \tag{19}$$

We observe two terms that are added and two that are subtracted. In the subtraction the limit is trivial and Equation (19) becomes:

$$\frac{\mu_0 c^2 q}{4\pi\gamma^2}\left\{\frac{2(x-v_x t)}{[(x-v_x t)^2]^{\frac{3}{2}}}\right\}\Delta_y\Delta_z \quad . \tag{20}$$

Then, the electric field divergent becomes:

$$\vec{\nabla}\cdot\vec{\mathrm{E}} = \frac{1}{\Delta_x\Delta_y\Delta_z}\frac{2\mu_0 c^2 q\Delta_y\Delta_z}{4\pi\gamma^2(x-v_x t)^2} = \frac{\mu_0 c^2 q}{4\pi\gamma^2(x-v_x t)^2\frac{\Delta_x}{2}} \quad . \tag{21}$$

For the charge to remain always inside the volume $dv = \Delta_x\Delta_y\Delta_z$, the distance between the charge and the point must be infinitesimal, that is, $(x - v_x t) \propto \Delta_x$. Considering the surface that limits the charge always on the border of this distance, then, the electric field center, which is the charge location, must be halfway of this distance that is $\frac{\Delta x}{4}$, as represented in Figure 4, thus, $(x - v_x t) = \frac{\Delta x}{4}$.

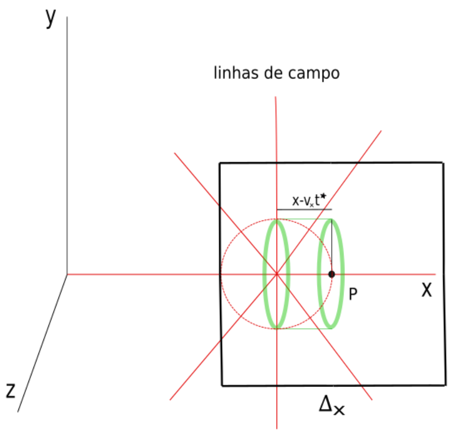

Figure 4 - Charge interpretation in Equation (22), the distance from point P up to the charge center is $x - v_x t^*$. The red circumference is the mental image that we are used to assuming as the region that limits the charge, only half of this circumference is observed to be within the cylinder with height and ray $x - v_x t^*$.

In this condition, Equation (21) recovers Gauss's law:

$$\vec{\nabla} \cdot \vec{\mathrm{E}} = \frac{\mu_0 c^2 q}{4\pi\gamma^2 \left(\frac{\Delta x}{4}\right)^2 \frac{\Delta x}{2}} = \frac{\mu_0 c^2 q}{\pi\gamma^2 \left(\frac{\Delta x}{2}\right)^2 \frac{\Delta x}{2}} = \frac{\rho'}{\gamma \varepsilon_0} = \frac{\rho}{\varepsilon_0} \quad . \tag{22}$$

Where $\pi\left(\frac{\Delta x}{2}\right)^2 \frac{\Delta x}{2}$ is the volume of a cylinder of base area $\pi\left(\frac{\Delta x}{2}\right)^2$ and length $\frac{\Delta x}{2}$, that is, the volume in which the charge is distributed is shaped as a cylinder.

The magnetic field divergence in Equation (12) is null for any value of (x, y, z).

2.2 AMPÈRE-MAXWELL'S LAW

Also, for the fields given by equations (10) and (12), we could observe that the Ampère-Maxwell's law is in fact satisfied:

$$\varepsilon_0 \mu_0 \frac{\partial \vec{E}(x,y,z;t)}{\partial t} = \vec{\nabla} \times \vec{\mathrm{B}}(x,y,z;t) =$$

$$= -\frac{1}{4} \frac{(v_x{}^2 - c^2)\mu_0 q v_x c\left(2c^2(v_x t - x)^2 + (y^2 + z^2)(v_x{}^2 - c^2)\right)}{\pi\left(c^2(v_x t - x)^2 - (y^2 + z^2)(v_x{}^2 - c^2)\right)^{\frac{5}{2}}} \hat{\imath}$$

$$+ \frac{3}{4} \frac{(v_x{}^2 - c^2)\mu_0 q v_x (v_x t - x) c^2}{\pi\left(c^2(v_x t - x)^2 - (y^2 + z^2)(v_x{}^2 - c^2)\right)^{5/2}} (y\hat{\jmath} + z\hat{k}) \quad . \tag{23}$$

It seems relevant to observe that Equation (23) is valid for any value of (x, y, z), at any point.

2.3 FARADAY'S LAW

Faraday's law is also satisfied by the fields chosen, that is, the rotational of the electric field at point P is equal to the negative of the magnetic field variation in time:

$$\vec{\nabla}\times\vec{E}(x,y,z;t) = -\frac{\partial\vec{B}(x,y,z;t)}{\partial t} = \frac{3v_x^2c^3}{4\pi}\frac{[\mu_0 q(v_x^2-c^2)](v_xt-x)(z\hat{\jmath}-y\hat{k})}{[c^2(v_xt-x)^2-(y^2+z^2)(v_x{}^2-c^2)]^{\frac{5}{2}}} \quad . \tag{24}$$

2.4 DISPLACEMENT CURRENT AND CHARGE CURRENT CASES

Since Equation (23) is valid for any point in space, we can now examine what happens if point P is placed on the charge trajectory. For $y = z = 0$, Equation (20) becomes:

$$\varepsilon_0\mu_0\frac{\partial\vec{E}(\vec{r},t)}{\partial t} = \frac{\mu_0}{2\pi}\frac{qv_x}{\gamma^2(x-v_xt)^3}\hat{\imath} \quad . \tag{25}$$

Equation (25) shows that the term $\pi(x-v_xt)^3$ can be interpreted as the volume of a ray cylinder of base equal to height $(x-v_xt)$, as represented in Figure 4. Since half of the field lines cross (half the flux) the cylinder of volume $V = \gamma^2\pi(x-v_xt)^2(x-v_xt) = \frac{V'}{\gamma}$, the term $\frac{q/2}{\gamma^2\pi(x-v_xt)^2(x-v_xt)}$ is, at the limit $(x-v_xt)\to 0$, then, the charge density is $\rho = \gamma\rho' = \frac{q/2}{\gamma^2\pi(x-v_xt)^2(x-v_xt)}$ at the referential without line. That is,

$$\varepsilon_0\mu_0\frac{\partial\vec{E}(\vec{r},t)}{\partial t} = \mu_0\rho v_x\hat{\imath} = \mu_0\vec{J} \quad . \tag{26}$$

Then, when the charge "touches" point P, Equation (26) is valid and we reach the surprising result that is: The displacement current term recovers the charge current density! After that, the displacement current term generalizes the charge current term and the Ampère-Maxwell's law can be written for all the regions in the space as:

$$\vec{\nabla}\times\vec{B}(x,y,z;t) = \frac{1}{c^2}\frac{\partial\vec{E}(x,y,z;t)}{\partial t} \quad , \tag{29}$$

it seems relevant to point out the similarity with the Faraday's law:

$$\vec{\nabla}\times\vec{E}(x,y,z;t) = -\frac{\partial\vec{B}(x,y,z;t)}{\partial t}. \tag{30}$$

3. RESULT ANALYSIS

This paper presented the problem of the punctiform charge dislocating at uniform speed in relation to a reference at rest. With the Lorentz electromagnetic four-potential transformations that obey Maxwell's equations, we obtained the electric and magnetic fields in a fixed position in relation to the resting referential. In this task, we considered that the Maxwell's equations are physics laws and that they do not change their form from one inertial reference to another, as required by Einstein's first postulate in the special relativity theory. Then, we submitted these fields to the relation that the Maxwell's equations impose, and we found, as the first result, that these fields obey Maxwell's equations and following this path, other interesting results were observed:

The second result to be highlighted in this study is that the displacement current term in the Ampère-Maxwell's law generalizes the "charge" current term, and, at the same time, the Ampère-Maxwell's law shows a closer symmetry with the Faraday's law.

The third result regards the interpretation of the electric charge that appears in the Coulomb's law, that is, the charge value is a scalar associated to the intensity of the electric field vector and the charge location is the location of the electric field center. Setting a limit for the charge volume means creating a charge concept, that is, something beyond the existing fields within a region and that do not exist outside that region. This concept, or this information, is not present in the fields given by Equations (10) and (12), however, we had to ascribe the charge an infinitesimal volume, which tends to zero. Therefore, in this study, we did not have to think the charge as a particle that has a finite volume, we could refer to the electric field intensity and the location of the electric field center only.

These results show that the displacement current term, which was "strange" in the Ampère-Maxwell's equation, becomes understandable, that is, it is a term that translates the relative movement of some electric field center, between this center and the point that is observed in the term. At the same time, the current density term that was normal, or well understood, was incorporated to the displacement current.

4. CONCLUSION

We know that the speed of the charge carriers in a metallic wire are very small ($\sim cm/h$), we also know that the Ampère's law was obtained from experimental observations in the case of wire currents, that is, the non-relativistic case. In fact, a relativistic treatment is not necessary to recover the Ampère-Maxwell's law, it is only necessary to assume Coulomb's electric field and Biot-Savart's electric field. However, in such case, the Faraday's law is only satisfied for the charge at rest. This "problem" is solved when we admit the relativistic case, even for low speeds. The movement of a punctiform "charge" generates the electric and magnetic field rotational and also the time variation of these fields, and these quantities relate to each other respecting the Maxwell's equations.

A point that deserves attention is the usual treatment given to a current in a wire. In this case, we considered a continuous distribution of charges that move toward the wire, in which the displacement current term calculation, at a point inside this wire, does not capture the electric field variation in time, since there is an assumption that no space exists between one charge and another. Thus, the part of the field that reduces due to the charges getting away from a point P is exactly the same that is replaced by the charges that get closer to the point. Outside the wire, the Coulomb's electric field, for the same reason, does not vary in time either. Therefore, the results presented in this study could not be reached by considering the charge continuous distribution hypothesis. In addition, the electric field outside the wire is usually assumed to be null. This is due to the presence of the same number of negative and positive charges inside the conductor. However, the field, due to presenting positive charges in a fixed position in relation to the wire, does not vary with time and presents a null rotational. Thus, the contribution to a time variation and also the existence of a rotational are due to the movement of negative charges.

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